

TECHNOLOGY ENRICHED INSTRUCTION IN YEAR 9 ALGEBRA¹

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This paper describes a pilot study involving the use of a computer treatment and a calculator treatment in a unit on linear equation solving in year 9. The computer treatment made use of spreadsheets, while the calculator treatment paralleled the computer treatment but used either previously prepared tables or ones constructed by the student with the aid of a calculator. Although both groups were able to generate tables to solve an equation using guess and check, computer treatment students did not discover on their own how to make full use of the spreadsheet capabilities to refine their guess and check approach.

INTRODUCTION

This is a report of a teaching experiment involving the use of a computer spreadsheet in a Year 9 unit on linear equation solving. The experiment was conducted in a Melbourne metropolitan secondary college during a four week period in 1992. The instructional material developed for the experiment was an adaptation of the computer graphic and numeric equation solving methods which were used in a year 12 equivalent bridging course in mathematics at the University of Melbourne (Asp, 1991). In this paper only results from the spreadsheet component are reported.

The intention of the study was to identify difficulties and problems associated with the classroom use of the material under conditions reflecting typical school and classroom organizational structures and teaching methodology. The material will be revised prior to engaging in further teaching experiments planned for the second semester of 1992. It was felt that through the use of the computer spreadsheet and graphing applications students might gain an understanding of algebraic concepts and techniques even though their skill in the manipulative aspects of algebra were not highly developed. Therefore, the study also sought to determine the effect of instructional treatment upon student learning of some important algebraic ideas.

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IMPORTANCE

Current mathematics curriculum statements within Australia and overseas are encouraging teachers to incorporate the use of computer and calculator technology into the teaching of mathematics.

...students should have sufficient experience of calculators and computers as mathematical tools to be able to make informed decisions about whether or not to use them in particular mathematical situations, and to use them efficiently when they wish to do so (A National Statement on Mathematics for Australian Schools, 1990, p. 14)

...we believe that students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems ... tools that simplify, but do not accomplish, the work at hand ... (Curriculum and Evaluation Standards for School Mathematics, 1989, p. 8)

In Victoria, year 11 and 12 mathematics students are required to use computer technology in their learning of mathematics (VCE Mathematics Study Design, 1989, p. 6). In addition, the nature of recent external assessment tasks at year 12 indicate that the ability to use common computer tools such as spreadsheet and graphing applications would be advantageous. It is too late to leave the development of an appropriate skill and knowledge base in the use of suitable computer tools such as spreadsheet and graphing applications until year 11 or 12. It is better that students become aware of such software in the early years of the secondary school. Hopefully, they will then be in a better position to use the technology as a "cognitive tool" which is not only capable of assimilation into established modes of thinking but also able to provide a medium for thinking and learning (Ruthven, 1992).

Even though the use of computer technology has potential for assisting students to learn mathematics and teachers are being encouraged to use this technology, few have taken up the challenge. As noted by Arnold:

Mathematics teachers, in particular, seem to have let a golden opportunity slip through their fingers. As the early 'owners' of the computers in schools, they failed to build upon this potential, content to focus upon programming (usually in BASIC) and 'drill and practice' software. The challenge was taken up by others - so much so that it tends to be the Social Science departments, the English departments, and others who dominate the use of 'the computer room' in many high schools today. (1992, p. 32)

Some of the reasons for this situation are lack of teacher knowledge of the technology and its potential use for the learning of mathematics, lack of suitable instructional materials to allow them to incorporate the technology into their teaching, lack of time to undertake explorations into these areas, and lack of facilities.

Research evidence is needed which documents the advantages and possible disadvantages to the learning of mathematics that both computers and calculators may provide. In addition, learning activities that have been proven by research to help students gain fundamental understandings while developing knowledge and skills in the use of this technology need to be developed and communicated to teachers.

THE NATURE OF THE STUDY

The study took place in a northern Melbourne metropolitan secondary college which had recently set up a laboratory of Macintosh computers. All four year 9 teachers agreed to be involved in the pilot study; none had previously used Macintosh computers in their classroom teaching. Two teachers had two year 9 classes, giving six classes in total. These two teachers each taught both a computer and a calculator class, enabling some control of the "teacher variable".

Two parallel teaching treatments for a unit on linear equation solving were developed, one involving the use of the computer and the other the use of a calculator. On the basis of access to the computer facilities, teacher desire to use computers, and the need for those teachers having two classes to have one assigned to each treatment, three classes were assigned to the computer treatment and three classes to the calculator treatment.

A student booklet of written instructions with exercises was produced for each treatment. Each treatment consisted of 15 class lessons with 7 lessons common to both treatments. Six of the common lessons involved the typical formal work with algebraic representation and equation solving found in year 9 algebra. The other common lesson was used to introduce a problem context for the equation solving lessons for each treatment. For the eight remaining lessons in each treatment, five involved the use of tables (spreadsheet) and three the use of linear graphs (graphing application).

The computer treatment made use of the spreadsheet application MS Works and the graphing application ANUGraph to assist students in solving linear equations and hopefully to help them draw links between symbolic, numeric and graphical equation solving. The calculator treatment students also solved equations from tables and graphs which were either constructed by the student with the aid of their calculator as a computational tool or given directly to the student.

An example of numeric equation solving using a spreadsheet is shown in Table 1. The student uses the spreadsheet to generate tables involving $F(x)$ and $G(x)$ by changing the step number or the initial value for x (or both) to solve the equation $F(x) = G(x)$ by a guess and check approach. Each time an initial or step value is entered the spreadsheet automatically recalculates to give a new table. Each spreadsheet was set up in the computer and the student simply accessed it.

Table 1: Spreadsheet for Numeric Equation Solving

Step	x	$F(x) = 4 + 0.25x$	$G(x) = 8 + 0.15x$	$F(x) - G(x)$
10	0	4	8	-4
	10	6.5	9.5	-3
	20	9	11	-2
	30	11.5	12.5	-1
	40	14	14	0
	50	16.5	15.5	1
	60	19	17	2
	70	21.5	18.5	3
	80	24	20	4
	90	26.5	21.5	5
	100	29	23	6

An example of graphic equation solving using a graphing application is shown in Figure 1. Here the student uses the application to draw the graphs for $y = F(x)$ and $y = G(x)$ and solves the equation $F(x) = G(x)$ by reading and interpreting the coordinates of the point of intersection.

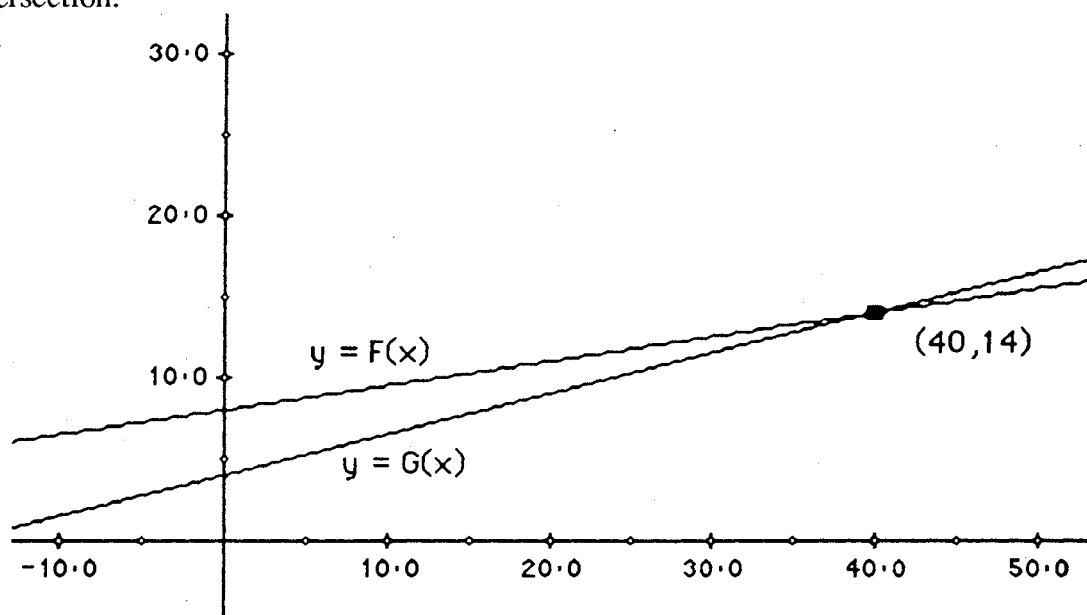


Figure 1: Line Graphs for Graphic Equation Solving

Figure 2 summarises the approach to equation solving used in the two treatments. As well as solving equations, students in each treatment were also asked to find rules which were used to generate a table of values. In the computer treatment students were encouraged to create different tables with the spreadsheet to assist them in finding the rule, and for the calculator treatment, students simply worked from given tables to find the rule. In addition, computer treatment students were asked to use a spreadsheet to create tables in order to compare two expressions and determine if they were sometimes, always or never equal. Calculator treatment students were asked to do the same but they used a table of values which they created by direct substitution into the expressions. The idea of sometimes, always or never was also approached through examination of graphs generated from two

given expressions with the computer drawing the graphs (computer treatment) or the students producing one of the two graphs to be compared (calculator treatment).

EQUATION SOLVING

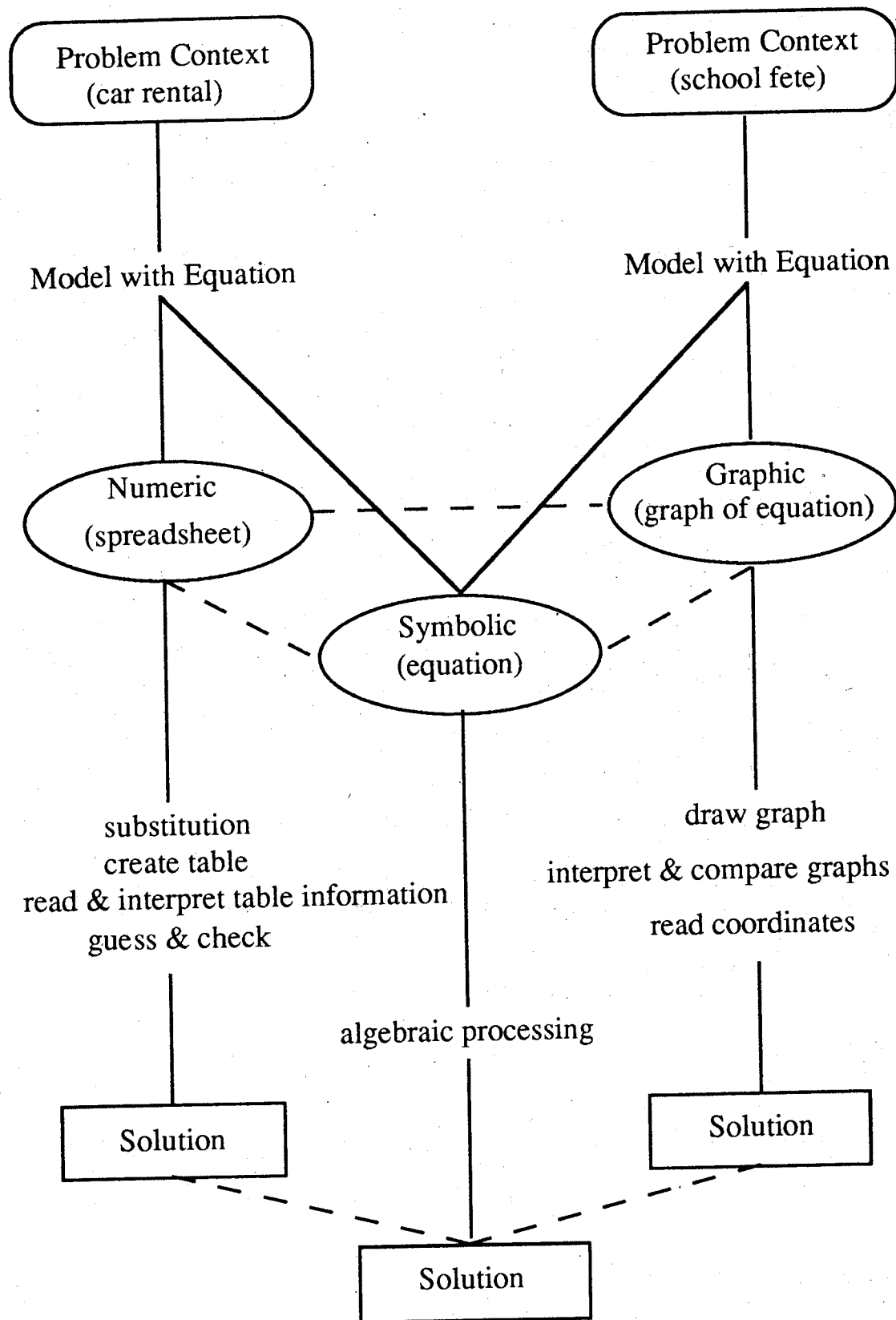


Figure 2: Summary of Approach to Equation Solving

ANTICIPATED OUTCOMES

It was hoped that computer treatment students could learn how to use the two computer applications to find the solution of equations and explore relationships between algebraic expressions in the time available and, in doing so, they would develop their understanding related to each of the following:

1. A letter is used to represent a variable number.
2. An expression is a variable number, dependent upon the value of the variable in which it is defined.
3. An equation is a statement that the numbers named by two expressions are sometimes (but not always) equal.
4. An identity is a statement that the numbers named by two expressions are always equal.
5. To solve an equation means to find the number(s) which make the two expressions which define the equation give the same number.
6. The solution(s) to an equation can be found by a guess and check approach and this process can be aided and refined by using relevant information in tables or spreadsheets.
7. The solution(s) to an equation can be found by a graphic approach by graphing the functions corresponding to each side of the equation and reading the coordinates of points of intersection of the graphs.

As the material for the calculator treatment paralleled that of the computer treatment it was felt that the above outcomes might apply equally to classes in the calculator treatment.

In order to gain some measure of the effectiveness of the treatments with respect to the anticipated outcomes each class was given a pre-test and post-test. Following the post-test, interviews were conducted with 12 students, two from each class with one at the 80th and one at the 40th percentile on the post-test. In addition, student worksheets were examined, observations were made of classes and, at the conclusion of the program, a meeting with the teachers was held to gain their views.

SOME RESULTS

The results of the written pre-tests and post-tests given to all students will be reported elsewhere. Each of the 12 students who were interviewed completed four tasks. A summary of the results for the first three of these interview tasks follows.

Task 1

The questions for Interview Task 1 are given in Figure 3.

- (a) x and y are numbers. $y = 8 + x$ and $x = 10$.
What is y ?
- (b) x and y are numbers. $y = 19 + (3x + 1)$ and $3x + 1 = 7$.
What is y ?
- (c) x and y are numbers. $y = 10 + (2x + 15)$ and $2x + 15 = 4$.
What is y ?

Figure 3: Task 1 Questions

The computer students did much better on this task than the calculator group with 5 of the 6 computer students answering all parts correctly. Only 3 of the 6 calculator group students answered part (a) correctly. One answered part (b) correctly and did so by finding $x = 2$ and then substituting, a method not used in the computer group. No calculator student answered part (c) correctly. A variety of syntax errors were made by the calculator students (e.g. " $19 + 4x = 23$ " for (b) and $y = 27$ for (c) because " $2x + 15 = 17x$ then add 10"). This question had been designed to probe whether students viewed an expression such as $2x + 15$ as a unit. Such stark differences between the two groups had not been expected. The corresponding item in the written test had been somewhat harder than the interview item (in effect only part (c) was given with no "lead-in") and had not shown a significant difference. This version of the item will be used in the next trial.

Task 2

The questions for Interview Task 2 are given in Figure 4.

In the algebra expression $5c + 1$, the letter c could stand for:

- | | | |
|-----------------------------------|-----|----|
| (a) 100 | YES | NO |
| (b) the number of kms travelled | YES | NO |
| (c) cars | YES | NO |
| (d) the number of cups on a table | YES | NO |
| (e) bottles | YES | NO |

Figure 4: Task 2 Questions

Students were asked to explain their answers and for each YES response were asked: "If c stood for that, what would $5c + 1$ stand for?". Three students (two computer and one calculator) believed that c could only be used if the initial letter of the relevant quantity or object was c . Four students (one computer and three calculator) avoided any choices which were objects. Two students (one from each group) said yes to all items but used numbers to explain the meaning of $5c + 1$. There seems to be a mismatch between the answers students give to the written item and their explanations of the meaning of $5c + 1$. Out of the 12, only two students read $5c + 1$ as other than a number (" 5 cars + 1" and " 5 cups + 1 cup"). Even though students seem to allow "letter as object" most were not confused by the meaning of $5c + 1$. Perhaps they would be more easily misled by the expression $5c$. This item needs work before it could be reliably used to diagnose student misconceptions in a written form.

Task 3

The context for Interview Task 3 is given in Figure 5 and the spreadsheet in Table 2.

The car rental context used in this item was similar to the problem context that the teaching materials had explored. Computer students used the spreadsheet to answer the questions, calculator students were provided with a table which could be used as a basis for interpolation.

All 12 students were able to correctly read $F(70)$ from the table or spreadsheet and 7 gave a satisfactory interpretation of its meaning in terms of renting a car from Mary or Joe. All 12 students were able to find x when $G(x) = 299.42$ (again a value from the table or spreadsheet) and 10 gave a satisfactory interpretation of its meaning. All 12 students were able to correctly find x from the table or spreadsheet when $F(x) - G(x) = 7.38$, but only 4 gave a satisfactory interpretation of its meaning.

Students were then asked to interpolate between the initially given values to find x when $F(x) = 323.50$ and to find the value of x which made the cost of renting equal, i.e. $F(x) = G(x)$. When solving the equation $F(x) = 323.50$, all of the calculator students were able to use the table to locate initial starting values of x correctly but only 2 were able to use their calculators to narrow down the answer to 87. Both groups of students preferred to compare $F(x)$ and $G(x)$ directly for the second part of the question, and only 2 students (both from the calculator group) referred directly to the data on $F(x) - G(x)$. All computer students confidently created new tables in an attempt to get 323.50, but there was no efficient use of start and step numbers. It seems that direct teaching will be required for students to appreciate strategies such as adjusting start numbers and step numbers alternatively to zoom in on an answer. Three of the 6 computer students found the correct solution (87) to $F(x) = 323.50$. Only 1 student (computer) found the correct solution (112) to $F(x) = G(x)$, although 1 calculator student used guess and check with calculator and gave 115 as his best answer.

Two questions required algebraic rearrangement before the spreadsheet or tables could be used. Three of the computer students and one calculator student correctly solved $F(x) = G(x) + 1.98$ (the solution could be read directly from the tables of $F(x) - G(x)$). Two computer students and one calculator student correctly solved $G(x) + 20 = 372.52$.

A person can rent a car from either Mary's Car Rentals or Joe's Car Rentals.

x is the number of kilometres the car is driven.

$F(x)$ is the cost of renting the car from Mary's Car Rentals and driving it x kilometres.

$$F(x) = 280 + 0.5x$$

$G(x)$ is the cost of renting the car from Joe's Car Rentals and driving it x kilometres.

$$G(x) = 269.92 + 0.59x$$

Figure 5: Context for Task 3

Table 2: Spreadsheet for Task 3

	Distance (kms)	Cost from Mary (dollars)	Cost from Joe (dollars)	
Step	x	$F(x)=280+0.5x$	$G(x)=269.92+0.59x$	$F(x)-G(x)$
10	10	285.00	275.82	9.18
	20	290.00	281.72	8.28
	30	295.00	287.62	7.38
	40	300.00	293.52	6.48
	50	305.00	299.42	5.58
	60	310.00	305.32	4.68
	70	315.00	311.22	3.78
	80	320.00	317.12	2.88
	90	325.00	323.02	1.98
	100	330.00	328.92	1.08
	110	335.00	334.82	0.18
	120	340.00	340.72	-0.72
	130	345.00	346.62	-1.62
	140	350.00	352.52	-2.52
	150	355.00	358.42	-3.42

COMMENTS

There is some evidence to suggest that students can learn to use a spreadsheet with guess and check to solve an equation. However, in their use of the computer students tended to concentrate on a single line of the spreadsheet, ignoring and failing to use other relevant information provided in the full spreadsheet display. Most students were unable to discover, on their own, the joint use of the step number and the starting number.

The instructional material required students to do too much in a short period of time. There was little time for them to discuss and reflect, something which would also have required more stimulus and direction from the teachers. Once students began to use the computers the teachers were reluctant to ask them to reflect on the process they were engaged in or on ideas related to this process. As a result, students tended to work only from the written instructional material with the teacher merely assisting this approach. More direct teacher input into the lessons was needed.

Despite these reservations, the teachers were enthusiastic about the use of computer spreadsheet and graphing applications in their classrooms, and they are planning to incorporate them into their future teaching of year 9 algebra. Student response was also generally quite positive.

The interview results have provided some measures of the differences between the computer and calculator groups. In particular, the computer groups seemed more able to see an expression as a single entity. This and other observations will form the basis for enhanced written pre- and post-tests which will be used to evaluate the success of a revised version of the student instructional materials and teaching approach.

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